ISSN: 2277-9655
(I2OR), Publication Impact Factor: 3.785
(ISRA), Impact Factor: 2.114

IJESRT

INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

PLANE GRAVITATIONAL WAVES IN BIMETRIC RELATIVITY

S. D. Deo*, Sulbha R. Suple
* Department of Mathematics N.S.Science and Arts College Bhadrawati,Dist-Chandrapur, (M.S.) 442902, India.
Department of Mathematics,Karmavir Dadasaheb Kannamwar College of Engineering, Nagpur 440009, India.

ABSTRACT

In this paper, Z = $\left[\left(x^1\right)^n+\left(x^2\right)^n+\ldots\ldots+\left(x^{n-1}\right)^n\right]^{\frac{1}{n}}-t$ and

$$Z=\left\{\frac{t}{\left[\left(x^1\right)^n+\left(x^2\right)^n+\ldots\ldots+\left(x^{n-1}\right)^n\right]^{\frac{1}{n}}}\right\}$$ type n-dimensional plane gravitational waves are studied with the source of gravitation Cosmic strings coupled with perfect fluid distribution and domain wall coupled with an electromagnetic source in Rosen's bimetric theory of relativity. It is shown that there is nil contribution from Comic strings coupled with perfect fluid and domain wall coupled with an electromagnetic source in this theory. Only vacuum model can be obtained.

KEYWORDS: Plane gravitational waves, Perfect fluid, Cosmic strings, Domain wall, Electromagnetic field ,Bimetric Relativity.
AMS Code-83C05 (General relativity)

INTRODUCTION

Though the general theory of relativity is one of the beautiful structure in all theoretical physics, some of the experts in the field have pointed out that there are some conceptual and physical difficulties which cannot be solved in general relativity. In order to get rid of the singular problems in general relativity Rosen [2] proposed a modification of the general relativity which is widely known as bimetric relativity(BR).

Rosen [2] has proposed at each point of space-time a Euclidean metric tensor γ_{ij} that describes the inertial forces in addition to the Riemannian metric tensor g_{ij} which interact with matter.With the flat background metric γ_{ij} the physical content of the theory is the same as that of the General relativity.

In this present work we have investigated the existence of plane gravitational wave with comic strings coupled with perfect fluid and domain wall coupled with electromagnetic field in bimetric relativity.

Therefore, we consider two line elements in modified theory of bimetric relativity are –

$$ds^2=g_{ij}dx^i dx^j \qquad (1.1)$$

And $$d\sigma^2=\gamma_{ij}dx^i dx^j \qquad (1.2)$$

Where ds is the interval between two neighboring events as measured by means of a clock and a measuring rod. The interval $d\sigma$ is an abstract or geometrical quantity not directly measurable.

One can regard it as describing the geometry that would exist if no matter were present. Using definition of plane wave, we will use here, Z=

$$\left[\left(x^1\right)^n+\left(x^2\right)^n+\dots\dots+\left(x^{n-1}\right)^n\right]^{\frac{1}{n}}-t$$ and

$$Z=\left\{\frac{t}{\left[\left(x^1\right)^n+\left(x^2\right)^n+\dots\dots+\left(x^{n-1}\right)^n\right]^{\frac{1}{n}}}\right\}$$ type n-dimensional plane gravitational waves

by using the line elements,

$$ds^2=-A\left(d[(x^1)^n]^2+d[(x^2)^n]^2+d[(x^3)^n]^2\right)$$
$$-C\sum_{i=4}^{n-1}d[(x^i)^n]^2+Adt^2 \qquad (1.3)$$

The theory of plane gravitational waves in general relativity has been introduced by many investigators like Einstein [1]; Takeno [3]; Bondi, Pirani andRobinson[4].

Takeno [5] has discussed the mathematical theory of plane gravitational waves and classified them into two categories, namely Z=(z-t) or (t/z)-type wave according as the phase function Z= (z-t) or (t/z)-type wave respectively. According to him, a plane wave g_{ij} is a non-flat solution of Ricci tensor $R_{ij}=0$ in general relativity and in some suitable coordinate system; all the component of the metric tensor are functions of a single variable Z=Z(z,t) (i.e. phase function).The theory of plane gravitational waves have been studied by many investigators,Lal and Ali[6]; Pandey et.al [8][9];Hogan P.A.[10]; Rane. R.S and Katore S. D (2009) [11],**;**Bhoyar S.R and Deshmukh A.G [12];Deo and Ronghe[13]; Deo and Suple [14] and they obtained the various solutions .

In this paper, we will study Z = $\left[\left(x^1\right)^n+\left(x^2\right)^n+\dots\dots+\left(x^{n-1}\right)^n\right]^{\frac{1}{n}}-t$ and

$$Z=\frac{t}{\left[\left(x^1\right)^n+\left(x^2\right)^n+\dots\dots+\left(x^{n-1}\right)^n\right]^{\frac{1}{n}}}$$

type n-dimensional plane gravitational waves with comic strings coupled with perfect fluid and domain wall coupled with electromagnetic field in bimetric relativity.

FIELD EQUATIONS IN BIMETRIC RELATIVITY

Rosen N. has proposed the field equations of Bimetric Relativity from variation principle as

$$K_i^j=N_i^j-\frac{1}{2}Ng_i^j=-8\pi\kappa T_i^j \qquad (2.1)$$

Where $$N_i^j=\frac{1}{2}\gamma^{\alpha\beta}\left[g^{hj}g_{hi|\alpha}\right]_{|\beta} \qquad (2.2)$$

$$N=N_\alpha^\alpha \quad , \quad \kappa=\sqrt{\frac{g}{\gamma}} \qquad (2.3)$$

and $$g=\det g_{ij} \quad ,\gamma=\det\gamma_{ij} \qquad (2.4)$$

Where a vertical bar (|) denotes a covariant differentiation with respect to γ_{ij} .

Z= $\left[\left(x^1\right)^n+\left(x^2\right)^n+........+\left(x^{n-1}\right)^n\right]^{\frac{1}{n}}-t$

TYPE PLANE GRAVITATIONAL WAVE WITH COSMIC STRINGS COUPLED WITH PERFECT FLUID

For Z= $\left[\left(x^1\right)^n+\left(x^2\right)^n+........+\left(x^{n-1}\right)^n\right]^{\frac{1}{n}}-t$ plane gravitational waves, we have the line element as

$$ds^2=-A\left(d[(x^1)^n]^2+d[(x^2)^n]^2+d[(x^3)^n]^2\right)$$
$$-C\sum_{i=4}^{n-1}d[(x^i)^n]^2+Adt^2 \tag{3.1}$$

where $A=A(Z)$, C = C (Z) and

Z = $\left[\left(x^1\right)^n+\left(x^2\right)^n+........+\left(x^{n-1}\right)^n\right]^{\frac{1}{n}}-t$

Corresponding to the equation (3.1),we consider the line element for background metric γ_{ij} a

$$d\sigma^2=-\begin{pmatrix}d[(x^1)^n]^2+d[(x^2)^n]^2+\\ d[(x^3)^n]^2+.......+d[(x^{n-1})^n]^2\end{pmatrix}+dt^2 \tag{3.2}$$

Since γ_{ij} is Lorentz metric (-1,-1,-1,-1,-1,……..-1, 1), Hence γ – covariant derivative becomes the ordinary partial derivative.

And, T_i^j the energy momentum tensor for cosmic strings coupled with perfect fluid is given by T_i^j =

$$T_{i\ string}^{j}+\left(\varepsilon+p\right)v_iv^j+pg_i^j \tag{3.3}$$

Where

$$T_{i\ string}^{j}=\rho v_iv^j-\lambda x_ix^j \tag{3.4}$$

Here ρ is the energy density for a cloud of cosmic strings with particle attached to them , λ the string tensor density, v_i -are the n-vectors representing the velocity of cloud of particles, x_i -the n-vectors representing an isotropic direction or say direction of strings and p *and* ε are proper pressure and matter density.The particle density associated with onfiguration is given by $\rho=\rho_p+\lambda$ where ρ_p is the particle density in the string cloud.

Moreover, the directions of strings satisfy the standard relations
$v_iv^j=-x_ix^j=1, v_ix^i=0\ if\ i\neq j$,we
consider the anisotropic direction along x-direction,

$$T_1^1=p+\lambda, T_2^2=T_3^3=........=T_{n-1}^{n-1}=p\ ,\ \ T_n^n=\varepsilon+2p+\rho$$

and T_i^j = 0 for i$\neq$j

Using equations (2.1) to (2.4) with (3.1) and (3.4), We get the field equations as

ISSN: 2277-9655
(I2OR), Publication Impact Factor: 3.785
(ISRA), Impact Factor: 2.114

$$D\left\{\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)+\frac{(n-4)}{2}\left(\frac{\bar{C}^2}{C^2}-\frac{\bar{\bar{C}}}{C}\right)\right\}=16\pi\kappa\left(p+\lambda\right) \quad (3.5)$$

$$D\left\{\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)+\frac{(n-4)}{2}\left(\frac{\bar{C}^2}{C^2}-\frac{\bar{\bar{C}}}{C}\right)\right\}=16\pi\kappa p \quad (3.6)$$

$$D\left\{\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)+\frac{(n-6)}{4}\left(\frac{\bar{C}^2}{C^2}-\frac{\bar{\bar{C}}}{C}\right)\right\}=8\pi\kappa p \quad (3.7)$$

$$D\left\{\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)+\frac{(n-4)}{2}\left(\frac{\bar{C}^2}{C^2}-\frac{\bar{\bar{C}}}{C}\right)\right\}=16\pi\kappa\left(\varepsilon+2p+\rho\right) \quad (3.8)$$

Where $D=\left\{\left(\frac{n-1}{n^2}\right)\left[\left(x^1\right)^n+\left(x^2\right)^n+\ldots\ldots+\left(x^{n-1}\right)^n\right]^{\frac{2(1-n)}{n}}-1\right\}$

and $\bar{A}=\frac{\partial A}{\partial Z}$, $\bar{\bar{A}}=\frac{\partial^2 A}{\partial Z^2}$, $\bar{C}=\frac{\partial C}{\partial Z}$, $\bar{\bar{C}}=\frac{\partial^2 C}{\partial Z^2}$ etc

Using equation (3.5) to (3.8), We get $p+\varepsilon+\rho-\lambda=0$ (3.9)

In view of the reality conditions i.e. (Hawking S.W.and Ellis G.F.R)[7]
$p>0,\lambda>0,\varepsilon>0 \text{ and } \rho>0$ must hold.

Using above conditions, the equation (3.9) is true only when

$$p=\varepsilon=\rho=\lambda=0 \quad (3.10)$$

Equation (3.10) immediately implies that cosmic strings coupled with perfect fluid does not exist in Z= $\left[\left(x^1\right)^n+\left(x^2\right)^n+\ldots\ldots\ldots+\left(x^{n-1}\right)^n\right]^{\frac{1}{n}}-t$ plane gravitational waves in Rosen's Bimetric theory of relativity and hence only vacuum model exists.Using (3.10), the field equations (3.5) to (3.8) reduces to the vacuum solutions

$$\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)=0 \quad (3.11)$$

and

$$\left(\frac{\bar{C}^2}{C^2}-\frac{\bar{\bar{C}}}{C}\right)=0 \quad (3.12)$$

Solving equations (3.11) and (3.12), we get

$$A=\beta e^{\alpha Z} \quad (3.13)$$

and

$$C=\gamma e^{\delta Z} \quad (3.14)$$

ISSN: 2277-9655
(I2OR), Publication Impact Factor: 3.785
(ISRA), Impact Factor: 2.114

where α, β, γ and δ are the constants of integration.

Thus substituting the value of (3.13) and (3.14) in (3.1), we get the vacuum line element as

$$ds^2 = -\beta e^{\alpha Z}\left(d[(x^1)^n]^2 + d[(x^2)^n]^2 + d[(x^3)^n]^2\right) - \gamma e^{\delta Z}\sum_{i=4}^{n-1} d[(x^i)^n]^2 + \beta e^{\alpha Z} dt^2 \tag{3.15}$$

By proper choice of co-ordinates, the metric (3.15) can be transform to

$$ds^2 = -e^{\xi Z}\begin{pmatrix} d[(x^1)^n]^2 + d[(x^2)^n]^2 + d[(x^3)^n]^2 \\ +\sum_{i=4}^{n-1} d[(x^i)^n]^2 - dt^2 \end{pmatrix} \tag{3.16}$$

which is free from singularity at t = 0 and the spatial volume of the model is given by

$$V^{n-1} = (-g)^{\frac{1}{2}} = (-1)^{\frac{n}{2}} e^{\frac{n\xi Z}{2}} \tag{3.17}$$

$$Z = \left\{ \frac{t}{\left[\left(x^1\right)^n + \left(x^2\right)^n + \dots\dots + \left(x^{n-1}\right)^n\right]^{\frac{1}{n}}} \right\}$$

TYPE PLANE GRAVITATIONAL WAVE WITH DOMAIN WALL COUPLED WITH AN ELECTROMAGNETIC SOURCE

For $Z = \left\{ \frac{t}{\left[\left(x^1\right)^n + \left(x^2\right)^n + \dots\dots + \left(x^{n-1}\right)^n\right]^{\frac{1}{n}}} \right\}$ plane gravitational waves, we have the line element as defined in (3.1) and (3.2)

And, T_i^j the energy momentum tensor for domain wall coupled with electromagnetic source is given by

$$T_i^j = T_i^j{}_{wall} + E_i^j \tag{4.1}$$

Where $$T_i^j{}_{wall} = \rho\left(g_i^j + \omega_i\omega^j\right) + p\omega_i\omega^j \tag{4.2}$$

with $\omega_i\omega^j = -1$ where ρ is the energy density of the wall , p is the pressure in the direction normal to the plane of the wall and ω_i is a unit space like vector in the same direction.

And $$E_i^j = F_{ir}F^{jr} - \frac{1}{4}F_{ab}F^{ab}g_i^j \tag{4.3}$$

ISSN: 2277-9655
(I2OR), Publication Impact Factor: 3.785
(ISRA), Impact Factor: 2.114

Where $E_i^{\ j}$ is the electromagnetic energy tensor, F_{ij} the electromagnetic field tensor.The magnetic field is taken along z-direction, so that the only non-zero component of F_{ij} is $F_{12} = -F_{21}$

Maxwell's equation is given by

$$F_{ij,\,k} + F_{jk,\,i} + F_{ki,\,j} = 0 \tag{4.4}$$

gives rise to $F_{12} = -F_{21} = F$ (a constant). (4.5)

Using equations (2.1) to (2.4) with (4.1) and (4.5), We get the field equations as

$$D\left\{\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)+\frac{(n-4)}{2}\left(\frac{\bar{C}^2}{C^2}-\frac{\bar{\bar{C}}}{C}\right)\right\}=16\pi\kappa\left(-p+\eta\right) \tag{4.6}$$

$$D\left\{\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)+\frac{(n-4)}{2}\left(\frac{\bar{C}^2}{C^2}-\frac{\bar{\bar{C}}}{C}\right)\right\}=16\pi\kappa\left(\rho+\eta\right) \tag{4.7}$$

$$D\left\{\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)+\frac{(n-6)}{4}\left(\frac{\bar{C}^2}{C^2}-\frac{\bar{\bar{C}}}{C}\right)\right\}=8\pi\kappa\left(\rho-\eta\right) \tag{4.8}$$

$$D\left\{\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)+\frac{(n-4)}{2}\left(\frac{\bar{C}^2}{C^2}-\frac{\bar{\bar{C}}}{C}\right)\right\}=16\pi\kappa\left(\rho-\eta\right) \tag{4.9}$$

Where $D=\left\{\dfrac{(n-1)t^2-\left[\left(x^1\right)^n+\left(x^2\right)^n+\ldots\ldots+\left(x^{n-1}\right)^n\right]^{\frac{2}{n}}}{\left[\left(x^1\right)^n+\left(x^2\right)^n+\ldots\ldots+\left(x^{n-1}\right)^n\right]^{\frac{4}{n}}}\right\}$

Using equation (4.6) to (4.9), We get $p+\rho-2\eta=0$ *and* $\eta=0$ (4.10)

In view of the reality conditions (Hawking S.W.and Ellis G.F.R) [7] i.e. $p>0$ *and* $\rho>0$ must hold.
Using above conditions (4.10) is satisfied only when

$$p=\rho=0 \tag{4.11}$$

Equation (4.10) and (4.11) immediately implies that domain wall coupled with electromagnetic field does not exist in

$Z=\left\{\dfrac{t}{\left[\left(x^1\right)^n+\left(x^2\right)^n+\ldots\ldots\ldots+\left(x^{n-1}\right)^n\right]^{\frac{1}{n}}}\right\}$ plane gravitational waves in Rosen's Bimetric theory of

relativity and hence only vacuum model exists.Using (4.10) and (4.11), the field equations (4.6) to (4.9) reduces to the vacuum solutions

$$\left(\frac{\bar{A}^2}{A^2}-\frac{\bar{\bar{A}}}{A}\right)=0 \tag{4.12}$$

and

ISSN: 2277-9655
(I2OR), Publication Impact Factor: 3.785
(ISRA), Impact Factor: 2.114

$$\left(\frac{\overline{C}^2}{C^2}-\frac{\overline{\overline{C}}}{C}\right)=0 \qquad (4.13)$$

Solving equations (4.12) and (4.13), we get

$$A=R_1\, e^{S_1 Z} \qquad (4.14)$$

and

$$C=R_2\, e^{S_2 Z} \qquad (4.15)$$

where $\mathrm{R}_1,\mathrm{R}_2,\mathrm{S}_1$ *and* S_2 are the constants of integration.

Thus substituting the value of (4.14) and (4.15) in (4.1), we get the vacuum line element as defined in equation (3.16).

MATERIALS AND METHODS

The plane gravitational waves are discussed with cosmic strings coupled with perfect fluid and domain wall coupled with electromagnetic field in the context of Bimetric theory of relativity. We have used the field equations of Bimetric theory of relativity analogous to general theory of relativity. Here we have used two metric tensor ds^2 and $d\sigma^2$ and one of them is flat metric ie $d\sigma^2$ which is in field equation given by equation (2.1) but not in the matter T_i^j.

In general theory of relativity only one metric tensor is used i.e. ds^2.

RESULTS AND DISCUSSIONS

In the above model, the solutions to the field equations are obtained as a vacuum model. This vacuum model represents Robertson –Walker flat model, which expands uniformly along space directions with time. The rate of expansion depends on the signature of the parameter ξ.

CONCLUSION

In the study of

$$Z=\left[\left(x^1\right)^n+\left(x^2\right)^n+\ldots\ldots\ldots+\left(x^{n-1}\right)^n\right]^{\frac{1}{n}}-t \text{ and}$$

$$Z=\left\{\frac{t}{\left[\left(x^1\right)^n+\left(x^2\right)^n+\ldots\ldots\ldots+\left(x^{n-1}\right)^n\right]^{\frac{1}{n}}}\right\}$$

type N-dimensional plane gravitational wave, there is nil contribution from the matter cosmic strings coupled with perfect fluid as well as domain wall coupled with electromagnetic field in Bimetric theory of relativity respectively. It is observed that the matter cosmic strings coupled with perfect fluid as well as domain wall coupled with electromagnetic field cannot be a source of gravitational field in the Rosen's bimetric theory but only vacuum model exists.

ACKNOWLEDMENT

The authors are thankful to Dr. R. D. Giri, Prof. Emeritus, P.G.T.D. (Mathematics), R. T. M. N. U., Nagpur, India for his constant inspiration.

REFERENCES

[1] Einstein and N. Rosen, J.Franklin Institute, 223, 43 (1937).

[2] Rosen N. (1973) A bimetric theory of gravitation I Gen. Rela. Grav.04, 435-47.
[3] Takeno H. (1958) A comparison of plane wave solutions in general relativity with those in non-symmetric theory.Prog.Theor.Phys.20,267-276
[4] Bondi H. ; Pirani, F.A.E. and Robinson, I.(1959). Gravitational waves in general relativity III.Exact plane waves. Proc.Roy.Soc.Lond.A23, 25, 519-533
[5] Takeno H. (1961)The mathematical theory of plane gravitational waves in General Relativity. Scientific report of Research Institute for theoretical Physics, Hiroshima University, Hiroshima, Ken, Japan
[6] Lal and Ali, (1970) Tensor, NS, 21, 134
[7] Hawking S.W. and Ellis G.F.R (1973) The Large scale structure of Space –time, pp.88,Cambridge Uni.Press.
[8] Pandey, S. N. (1979) Plane wave solutions in Finzi's non-symmetric unified field theory. Theo. Math. Phys. 39, 371-375
[9] S.N.Pandey (1975) Plane wave solutions of Weakened Field Equations in Peres space-time, Tensor N.S., 29, 264.
[10] Hogan, P.A.(1999) Gravitational waves and Bertotti-Robinson space- time Math. Proc. Roy. Irish Acad. 99A, 51-55.
[11] Rane. R.S and Katore S. D (2009) Generalized Plane gravitational wave of Weakened Field Prespace-time Journal, Nov.2011, Vol. 2. Issue 11 1845-1851 general relativity, J.M
[12] Bhoyar S.R and Deshmukh A.G. (2012) Plane wave solutions of the field equations of general relativity –II Romanian Reports in Physics, Vol. 64, No.4, P. 933–944
[13] Ronghe A.K. and Deo S.D.(2011) Plane gravitational waves with wet dark energy.International Journal of MathematicalArchive- 2(3) Mar.-391-392
[14] Deo S.D. and Suple S.R. (2015) Plane gravitational waves with Cosmic Strings coupled with Maxwell's field in BimetricRelativity. IOSR-Journal of Mathematics, Volume 11, Issue 2,Ver.-III,(Mar-Apr. 2015) , PP 15-21